## Learning Individually Fair Classifier with Path-Specific Causal-Effect Constraint

Yoichi Chikahara ${ }^{1,3}$, Shinsaku Sakaue ${ }^{2}$, Akinori Fujino ${ }^{1}$, Hisashi Kashima ${ }^{3}$

Proposed method
Main idea: Make $Y_{A \leftarrow 0}=Y_{A \leftarrow 1 \| \pi}=0$ or $Y_{A \Leftarrow 0}=Y_{A \Leftarrow 1 \| \pi}=1$ for all individuals (i.e., regardless of input feature value $\boldsymbol{x}$ )

## 1. Penalty by upper bound on PIU

To achieve this goal, we force probability of individual unfairness (PIU) to be zero, whose upper bound can be derived as

$$
\begin{aligned}
& \mathrm{P}\left(Y_{A \Leftarrow 0} \neq Y_{A \Leftarrow 1 \| \pi}\right) \leq 2 \mathrm{P}^{I}\left(Y_{A \Leftarrow 0} \neq Y_{A \Leftarrow 1 \| \pi}\right) \\
& \mathrm{P}^{I}\left(Y_{A \leftarrow 0}, Y_{A \leftarrow 1 \| \pi}\right)=\mathrm{P}\left(Y_{A \Leftarrow 0}\right) \mathrm{P}\left(Y_{A \leftarrow 1 \| \pi}\right) \\
& \text { is an independent joint distribution, which can be inferred } \\
& \text { from data without any restrictive functional assumptions }
\end{aligned}
$$

To make the upper bound value close to zero, we use the estimator of $\mathrm{P}^{I}\left(Y_{A \Leftarrow 0} \neq Y_{A \leftarrow 1 \| \pi}\right)$ as penalty function, which is formulated as

$$
G_{\theta}\left(\boldsymbol{x}_{1}, \ldots, \boldsymbol{x}_{n}\right)=\hat{p}_{\theta}^{A \leftarrow 1 \| \pi}\left(1-\hat{p}_{\theta}^{A \leftarrow 0}\right)+\left(1-\hat{p}_{\theta}^{A \leftarrow 1 \| \pi}\right) \hat{p}_{\theta}^{A \leftarrow 0},
$$

where $\hat{p}_{\theta}^{A \leftarrow=}$ and $\hat{p}_{\theta}^{A \in 1 \| \pi}$ are estimator of $\mathrm{P}\left(Y_{A \Leftarrow 0}=1\right)$ and $\mathrm{P}\left(Y_{A \in 1 \| \pi}=1\right)$, In Example 1, they are given as weighted averages of $c_{\theta}(\boldsymbol{X})=\mathrm{P}(Y=1 \mid \boldsymbol{X})$ $\hat{p}_{\theta}^{A=0}=\frac{1}{n} \sum_{i=1}^{n} \mathbf{1}\left(a_{i}=0\right) \hat{\psi}_{i} c_{\theta}\left(a_{i}, q_{i}, d_{i}, m_{i}\right) \quad \hat{p}_{\theta}^{A=1 \| \pi}=\frac{1}{n} \sum_{i=1}^{n} 1\left(a_{i}=1\right) \hat{\psi}_{i}^{\prime} c_{\theta}\left(a_{i}, q_{i}, d_{i}, m_{i}\right)$

## 2. Comparison with existing fairness constraint

Our method aims to satisfy the following condition:

$$
\hat{p}_{\theta}^{A \rightarrow 1 \| \pi}\left(1-\hat{p}_{\theta}^{A \leftarrow=0}\right)+\left(1-\hat{p}_{\theta}^{A \in 1 \| \pi}\right) \hat{p}_{\theta}^{A \in 0} \leq \delta .
$$

By contrast, the existing FIO method [3] imposes the following one:

$$
-\delta^{\prime} \leq \hat{p}_{\theta}^{A \leftarrow 1 \| \pi}-\hat{p}_{\theta}^{A \Leftarrow 0} \leq \delta^{\prime}
$$

Figure 1: Feasible regions of our constraint (red) and FIO (blue)



| $Y_{A \in 0}=Y_{A \in 1 \\| \pi}=0$ or $Y_{A \in 0}=Y_{A \in=1 \\| \pi}=1$ <br> holds with high probability.$\quad$It is uncertain whether $Y_{A \in 0}$ and $Y_{A \in=1 \\| \pi}$ <br> take the same value. |
| :--- |

3. Extension for addressing latent confounders

Marginal probabilities $\hat{p}_{\theta}^{A}=0$ and $\hat{p}_{\theta}^{A \in 1 \| \pi}$ are difficult to estimate when there are unobserved variables called latent confounders. Nevertheless, if their lower and upper bounds are available, we can achieve individual-level fairness using the following penalty: $G_{\theta}\left(\boldsymbol{x}_{1}, \ldots, \boldsymbol{x}_{n}\right)=\hat{u}_{\theta}^{A \Leftarrow 1 \| \pi}\left(1-\hat{\imath}_{\theta}^{A \leftarrow 0}\right)+\left(1-\hat{l}_{\theta}^{A \leftarrow 1 \| \pi}\right) \hat{u}_{\theta}^{A \Leftarrow 0}$

## Experimental results

We compared our method with the following four baselines:

1. FIO [3]: constrains the expected value of PSEs
2. PSCF [4]: aims to reduce the conditional expected value of PSEs
3. Unconstrained: imposes no fairness constraint or penalty
4. Remove [5]: not use any features that are affected by sensitive feature

Table 2 and Figure 2 shows the test accuracy and the four statistics of unfairness: (i) the expected value of PSEs, (ii) the std. in conditional expected values of PSEs, (iii) Upper bound on PIU, and (iv) PIU.

| Method | Synth | German Adult | Proposed achieved comparable accuracy to PSCF. |
| :---: | :---: | :---: | :---: |
| Proposed | $80.0 \pm 0.9$ | $75.0 \quad 75.2$ |  |
| FIO | $84.8 \pm 0.6$ | 78.08181 .2 |  |
| PSCF | $74.8 \pm 1.6$ | $\begin{array}{ll}76.0 & 73.4 \\ 810 & 83\end{array}$ |  |
| Unconstrained | $88.2 \pm 0.9$ 76.9 | $\begin{array}{ll}81.0 & 83.2 \\ 73.0 & 74.7\end{array}$ |  | Remove

Figure 2: Four statistics of unfairness on test data


With Proposed, all unfairness With Proposed, all unfairness
statistics were close to zero. PSCF failed to reduce the std. in conditional expected values of PSEs (i.e., (ii)) because the data violates the functional assumptions.

According to Wu et al. [2], a classifier achieves (path-specific) individual-leve fairness if the conditional expected value of PSEs is zero for any input $\boldsymbol{x}$ :

$$
\left.\left.\mathbb{E}_{Y_{A \Leftarrow 0}, Y_{A \Leftarrow 1 \| \pi}} \frac{\left[Y_{A \Leftarrow 1 \| \pi}-Y_{A \Leftarrow 0}\right.}{} \right\rvert\, \boldsymbol{X}=\boldsymbol{x}\right]=0
$$

PSE: difference of two predictions (i.e., $Y_{A \Leftarrow 0}$ and $Y_{A \Leftarrow 1 \| \pi}$ ), obtained by modifying input feature attributes $\boldsymbol{x}$. In Example 1, for each woman $(A=0)$, $Y_{A \Leftarrow 0}$ is made by directly taking her observed feature attributes as input $Y_{A \Leftarrow 1 \| \pi}$ is made with counterfactual attributes, observed if she were male $(A=1)$

## How can we learn individually fair classifier without restrictive functional assumptions?

