Causal Inference in Time Series via Supervised Learning

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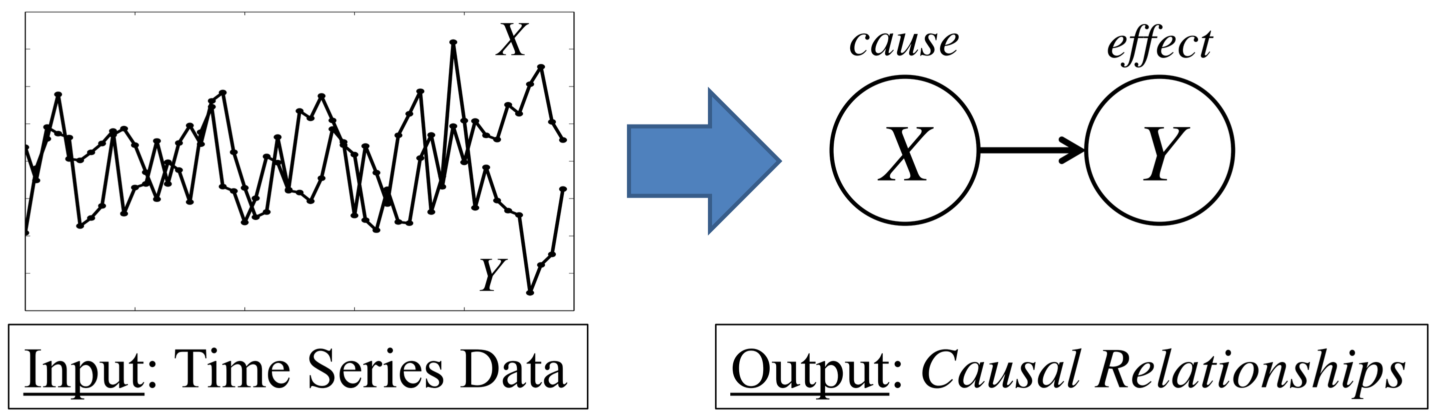
Problem Setting

- Causal inference in time series: A knowledge discovery task



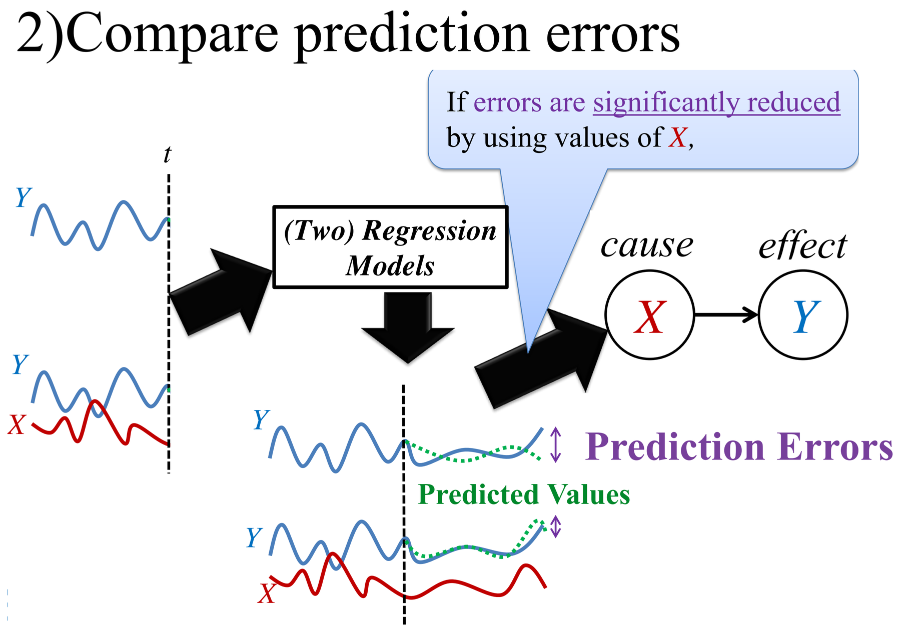
- Granger causality:

> X is the cause of Y
> if the past values of X are **helpful in predicting** the future values of Y

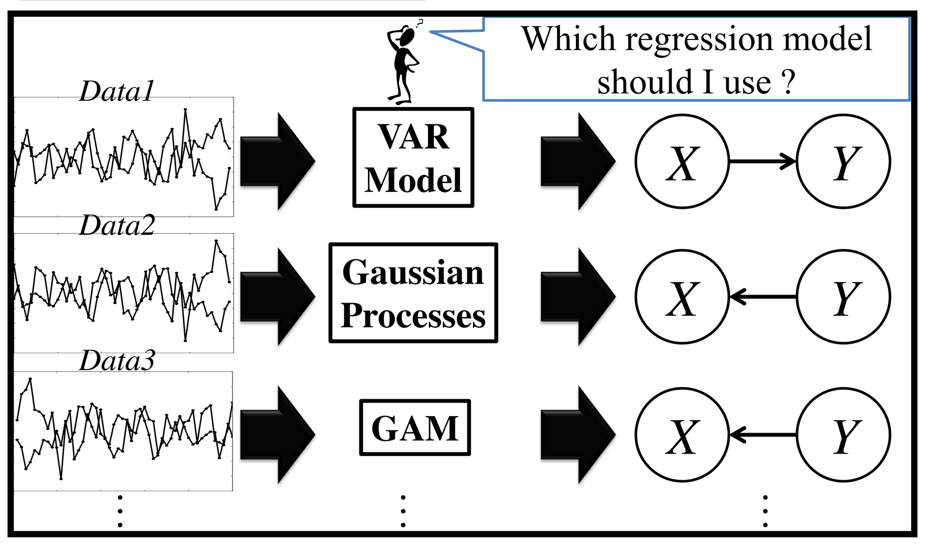
Weak Points in Existing Methods

Approach:

1)Fit (two) regression models
2)Compare prediction errors



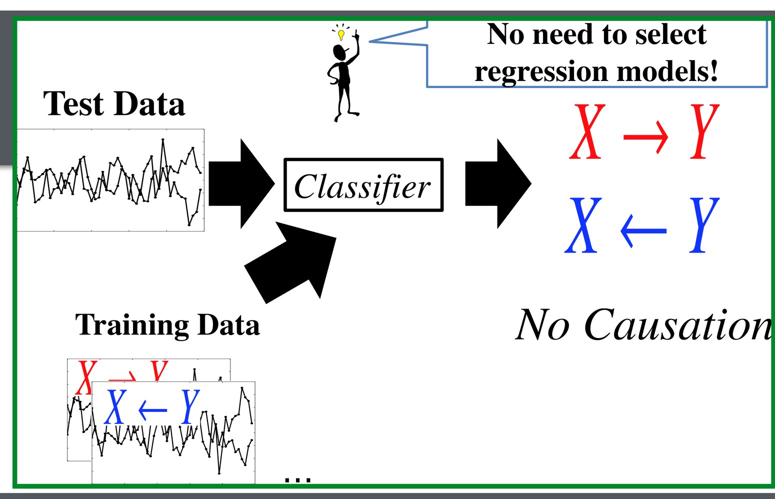
Weak Points:



Model misspecification leads to **low inference accuracy**

Main Contribution

Propose a **supervised learning approach** to Granger causality identification problem that **requires no selection of regression models**



Ideas for Classifier Design

Review Granger causality definition:

✓ X is the cause of Y

if $P(Y_{t+1}|S_X, S_Y) \neq P(Y_{t+1}|S_Y)$

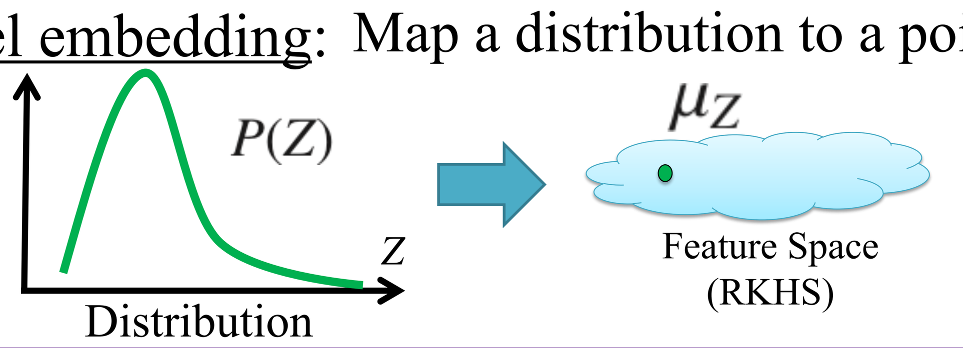
Distribution of Y_{t+1} given past values of Y and X $\neq$ Distribution of Y_{t+1} given past values of Y

✓ X is **not** the cause of Y

$P(Y_{t+1}|S_X, S_Y) = P(Y_{t+1}|S_Y)$

$S_X = \{x_1, \cdots, x_t\}$
$S_Y = \{y_1, \cdots, y_t\}$

Review Kernel embedding: Map a distribution to a point in feature space



A classifier that assigns labels by **following rule** can infer Granger causality

If $\begin{cases} P(Y_{t+1}|S_X,S_Y) \neq P(Y_{t+1}|S_Y) \\ P(X_{t+1}|S_X,S_Y) = P(X_{t+1}|S_X) \end{cases}$ then $X \rightarrow Y$

If $\begin{cases} P(Y_{t+1}|S_X,S_Y) = P(Y_{t+1}|S_Y) \\ P(X_{t+1}|S_X,S_Y) \neq P(X_{t+1}|S_X) \end{cases}$ then $X \leftarrow Y$

If $\begin{cases} P(Y_{t+1}|S_X,S_Y) = P(Y_{t+1}|S_Y) \\ P(X_{t+1}|S_X,S_Y) = P(X_{t+1}|S_X) \end{cases}$ then *No Causation*

By using kernel embedding, the above assignment rules are reformulated as

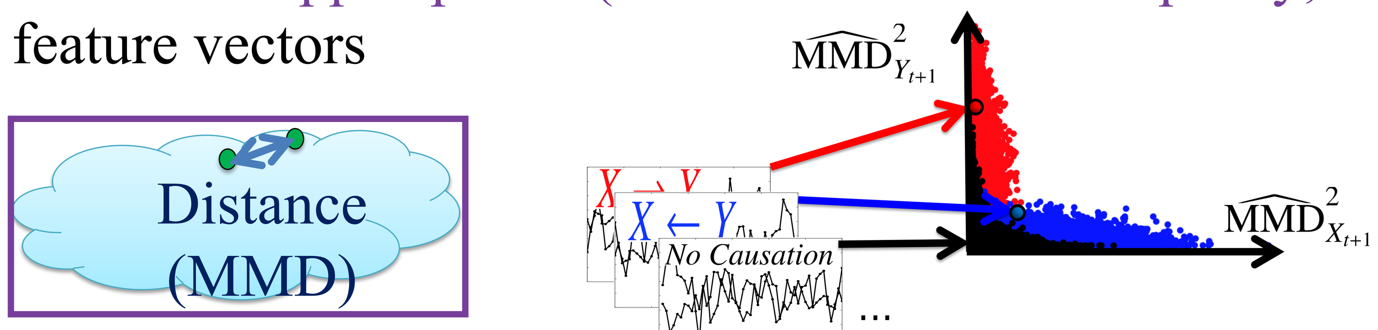
If $\begin{cases} \mu_{X_{t+1}|S_X,S_Y} = \mu_{X_{t+1}|S_X} \\ \mu_{Y_{t+1}|S_X,S_Y} \neq \mu_{Y_{t+1}|S_Y} \end{cases}$ then $X \rightarrow Y$

If $\begin{cases} \mu_{X_{t+1}|S_X,S_Y} \neq \mu_{X_{t+1}|S_X} \\ \mu_{Y_{t+1}|S_X,S_Y} = \mu_{Y_{t+1}|S_Y} \end{cases}$ then $X \leftarrow Y$

If $\begin{cases} \mu_{X_{t+1}|S_X,S_Y} = \mu_{X_{t+1}|S_X} \\ \mu_{Y_{t+1}|S_X,S_Y} = \mu_{Y_{t+1}|S_Y} \end{cases}$ then *No Causation*

Feature Representation

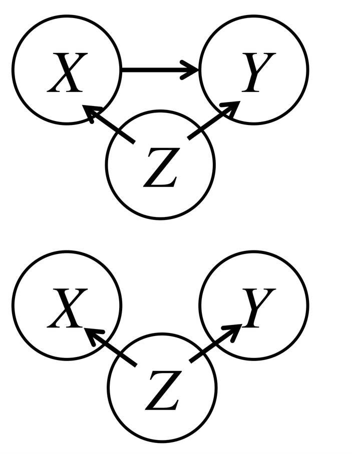
To design classifier based on the above ideas, we utilize the distance between mapped points (maximum mean discrepancy; MMD) to obtain feature vectors



We can expect estimated MMDs to be sufficiently different depending on causal directions

Extension to Multivariate Time Series

Review Conditional Granger causality: extended definition for multivariate case

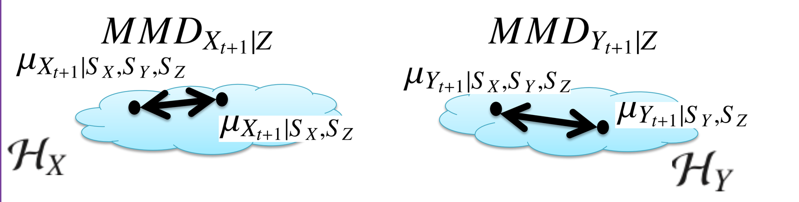


if $P(Y_{t+1}|S_X, S_Y, S_Z) \neq P(Y_{t+1}|S_Y, S_Z)$

Both distributions are given past values of *third variable* Z

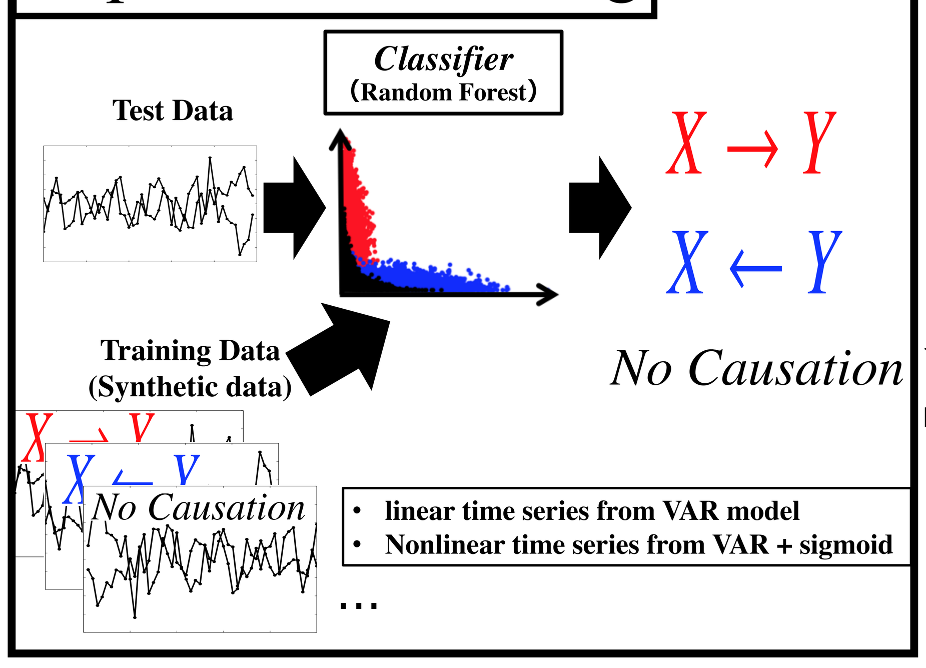
if $P(Y_{t+1}|S_X, S_Y, S_Z) = P(Y_{t+1}|S_Y, S_Z)$

Based on conditional Granger causality definition, we similarly estimate MMDs to obtain feature vectors for multivariate time series

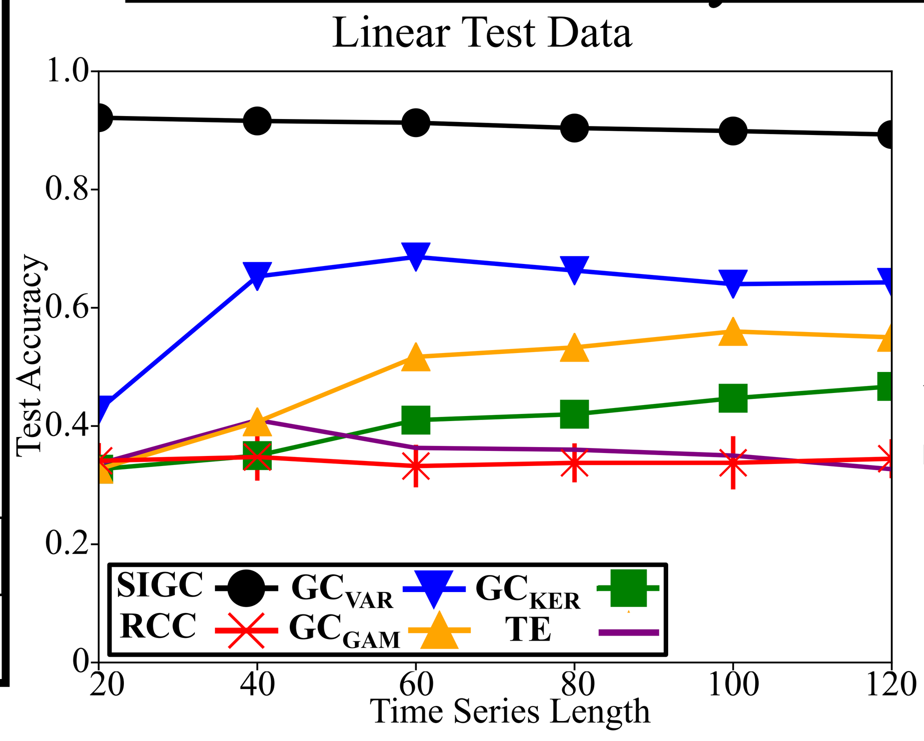


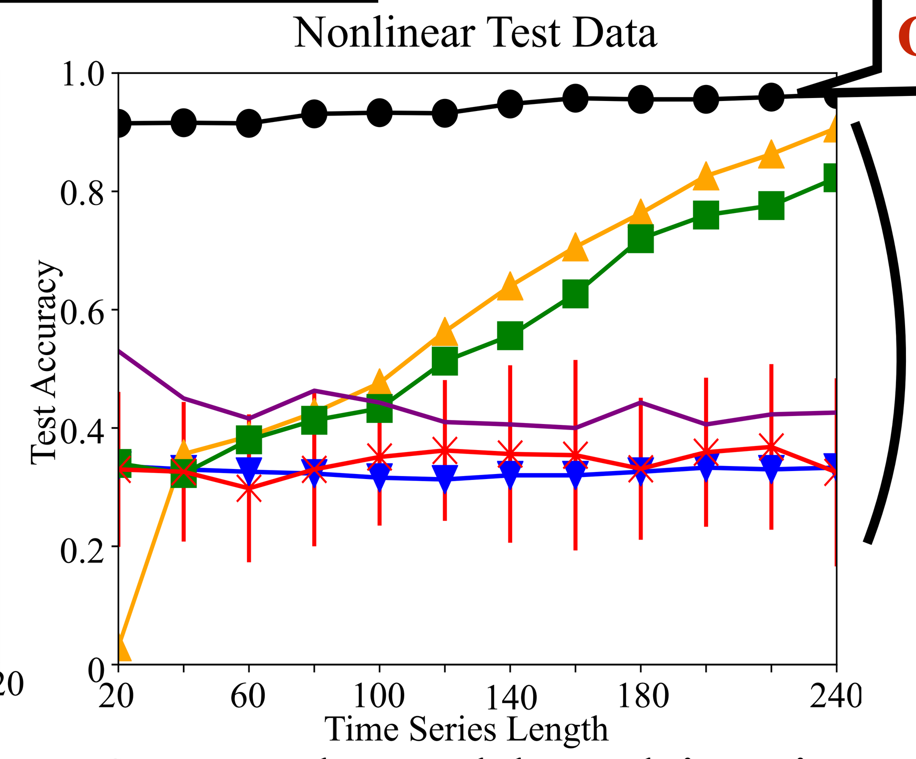
Experimental Results

Experimental Setting



1. Linear / Nonlinear synthetic time series





Our method worked sufficiently better

Existing methods worked poorly when regression models could not be well fitted to data

2. Real-world bivariate time series

e.g., *River Runoff*:
X: Precipitation
Y: River runoff
True causality: $X \rightarrow Y$

	SIGC	RCC	GC_{VAR}	GC_{GAM}	GC_{KER}	TE
Temperature ($T = 200$)	**0.961** (0.011)	0.432 (0.242)	0.950	0.848	0.234	0.492
Radiation ($T = 200$)	**0.987** (0.053)	0.515 (0.345)	0.156	0.0	0.782	0.394
Internet ($T = 200$)	**1.0** (0.0)	0.478 (0.222)	0.157	0.387	0.261	0.498
Sun Spots ($T = 200$)	**1.0** (0.0)	0.435 (0.182)	0.908	0.704	0.076	0.522
River Runoff ($T = 200$)	**0.958** (0.058)	0.399 (0.193)	0.684	0.406	0.155	0.485

3. Real-world multivariate time series

✓ Yeast cell cycle gene expression data (14 genes)

	$SIGC_{tri}$	$SIGC_{bi}$	RCC	GC_{VAR}	GC_{GAM}	GC_{KER}	TE
macro F1	**0.483** (0.0)	0.431 (0.007)	0.407 (0.096)	0.457	0.437	0.351	0.430
micro F1	**0.637** (0.0)	0.578 (0.011)	0.567 (0.161)	0.567	0.513	0.436	0.449